Exercise 28

Explain, using Theorems 4, 5, 7, and 9, why the function is continuous at every number in its domain. State the domain.

$$R(t) = \frac{e^{\sin t}}{2 + \cos \pi t}$$

Solution

The numerator is $e^{\sin t}$, a composition of two continuous functions by Theorem 7, which is continuous according to Theorem 9. The denominator is $2 + \cos \pi t$, a composition of two continuous functions by Theorem 7, which is continuous according to Theorem 9. $R(t)$ is the ratio of these functions, which according to Theorem 4 is also continuous wherever the denominator is not zero. Since $-1 \leq \cos \pi t \leq 1$, $2 + \cos \pi t$ is never zero. The domain is

$$(-\infty, \infty).$$